PRODUCTION SYSTEMS ENGINEERING

Chapter 5: Continuous Improvement of Bernoulli Lines

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- Motivation:
  - Continuous improvement is universally recognized as the most important managerial tool.
  - Currently, continuous improvement projects have no theory for their design and are, typically, carried out based on experience, common sense, and sometimes, discrete event simulations. As a result, quite often predicted improvements do not materialize.
  - The purpose of this chapter is to present analytical tools for designing continuous improvement projects with predictable results.
  - In this framework, both constrained and unconstrained improvabilities are addressed.

OUTLINE

1. Constrained Improvability
2. Unconstrained Improvability
3. Measurement-Based Management
4. Case Studies
5. Summary

1. Constrained Improvability

1.1 Resource constraints and definitions

- Constraints:

$$\sum_{i=1}^{M-1} N_i = N^* \qquad \textit{Buffer capacity (BC) constraint}$$

$$\prod_{i=1}^{N} p_i = p^* \qquad \textit{Workforce (WF) constraint}$$

1.1 Resource constraints and definitions (cont.)

- **Definition:** A Bernoulli line is
  - *improvable w.r.t. BC* if there exist $N_1', \ldots, N_{M-1}'$ s.t. $\sum_{i=1}^{M-1} N_i' = N^*$ and

$$\widehat{PR}(p_1, \ldots, p_M, N_1', \ldots, N_{M-1}') > \widehat{PR}(p_1, \ldots, p_M, N_1, \ldots, N_{M-1})$$

  - *improvable w.r.t. WF* if there exist $p_1', \ldots, p_M'$ s.t. $\prod_{i=1}^{M} p_i' = p*$ and

$$\widehat{PR}(p_1', \ldots, p_M', N_1, \ldots, N_{M-1}) > \widehat{PR}(p_1, \ldots, p_M, N_1, \ldots, N_{M-1})$$

  - *improvable w.r.t. BC and WF simultaneously* if the exist $N_1', \ldots, N_{M-1}'$ and $p_1', \ldots, p_M'$ s.t.

$$\sum_{i=1}^{M-1} N_i' = N^*, \qquad \prod_{i=1}^{M} p_i' = p*$$

and $\widehat{PR}(p_1', \ldots, p_M', N_1', \ldots, N_{M-1}') > \widehat{PR}(p_1, \ldots, p_M, N_1, \ldots, N_{M-1})$

1.2 Improvability w.r.t. WF

1.2.1 Necessary and sufficient conditions

Theorem: A Bernoulli line is unimprovable w.r.t. WF if and only if

$$p_i^f = p_{i+1}^b, \qquad i = 1, \ldots, M-1$$

Corollary: Under the above condition,

$$\widehat{WIP}_i = \frac{N_i(N_i+1)}{2(N_i+1-p_i^f)}, \qquad i = 1, \ldots, M-1$$

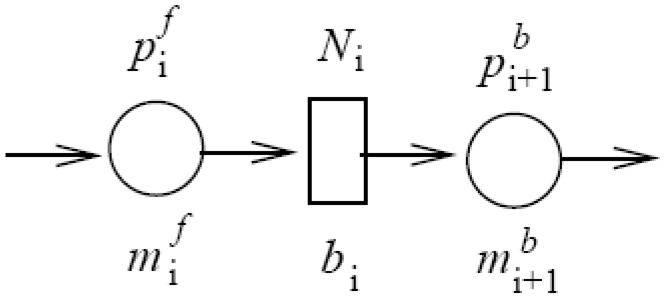
i.e.,

$$\frac{N_i}{2} < \widehat{WIP}_i < \frac{N_i+1}{2}, \qquad i = 1, \ldots, M-1.$$

1.2.2 WF improvability indicator and continuous improvement procedure

- **WF-Improvability Indicator:** A Bernoulli line is practically unimprovable w.r.t. WF if each buffer is close to being half full.

- “Common sense” justification of this indicator:



1.2.2 WF improvability indicator and continuous improvement procedure (cont)

- **WF-Continuous Improvement Procedure:**

  (1) Evaluate WIP_i

  (2) Determine the buffer for which $|WIP_i - N_i/2|$ is maximum

  (3) Re-allocate workforce

  (4) Return to (1)

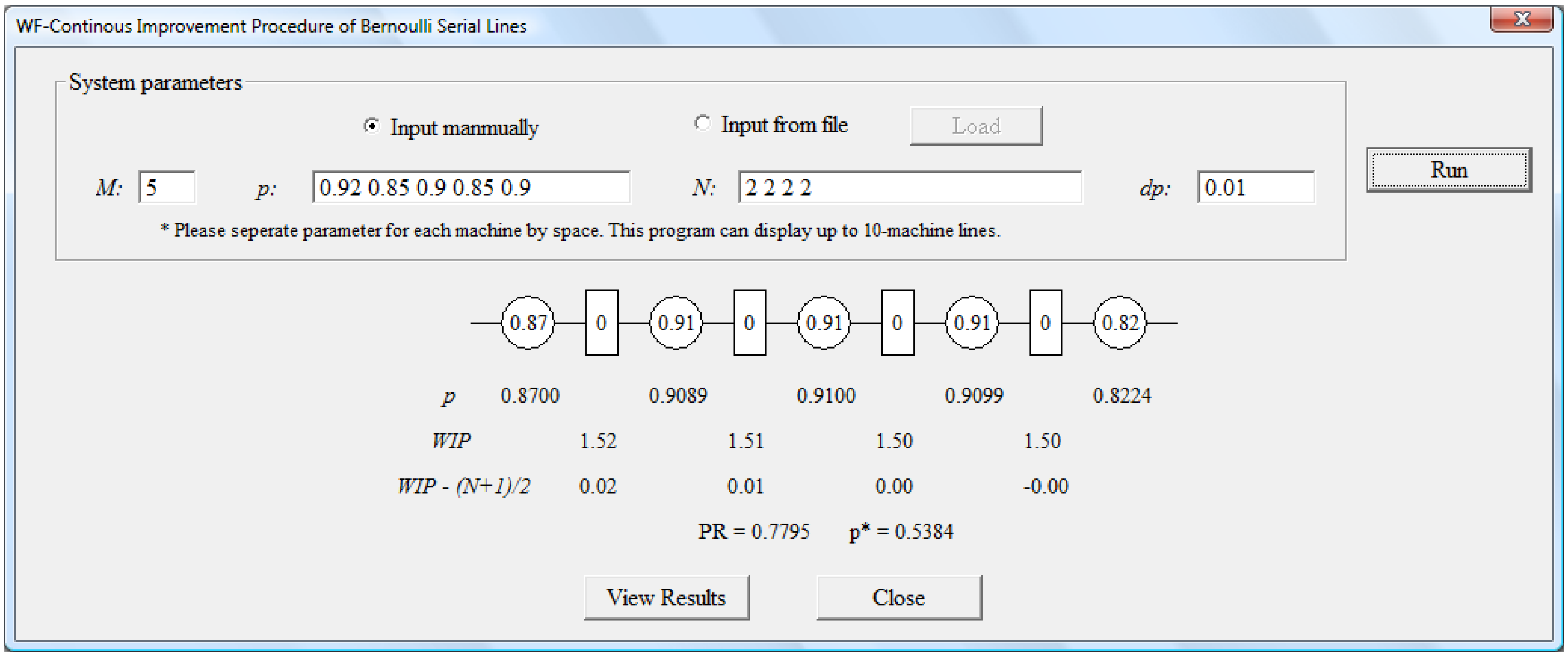
  (5) Continuous until $\max|WIP_i - N_i/2|$ is sufficiently small

1.2.3 Example

Step	p_i				$\lvert\widehat{WIP}_i - \frac{N_i}{2}\rvert$			$\widehat{PR}$
1	0.9675	0.9225	0.8780	0.8372	2.3074	1.8641	1.0855	0.8281
2	0.9578	0.9318	0.8780	0.8372	2.2322	1.9764	1.0936	0.8282
3	0.9482	0.9412	0.8780	0.8372	2.1478	2.0774	1.0984	0.8283
4	0.9388	0.9507	0.8780	0.8372	2.0531	2.1674	1.1010	0.8284
5	0.9388	0.9412	0.8869	0.8372	2.0430	2.0654	1.2504	0.8307
6	0.9388	0.9318	0.8958	0.8372	2.0346	1.9493	1.3948	0.8326
7	0.9294	0.9412	0.8958	0.8372	1.9236	2.0528	1.4000	0.8326
8	0.9294	0.9318	0.9049	0.8372	1.9150	1.9357	1.5398	0.8341
9	0.9294	0.9225	0.9140	0.8372	1.9085	1.8046	1.6698	0.8352
10	0.9201	0.9318	0.9140	0.8372	1.7811	1.9203	1.6783	0.8352
11	0.9201	0.9225	0.9232	0.8372	1.7753	1.7885	1.8004	0.8360
12	0.9201	0.9225	0.9140	0.8457	1.7165	1.7261	1.5854	0.8431
13	0.9201	0.9133	0.9232	0.8457	1.7084	1.5605	1.7103	0.8440
14	0.9201	0.9133	0.9140	0.8542	1.6448	1.4753	1.4606	0.8506
15	0.9109	0.9225	0.9140	0.8542	1.4690	1.6181	1.4833	0.8508
16	0.9109	0.9133	0.9232	0.8542	1.4550	1.4249	1.6123	0.8519
17	0.9109	0.9133	0.9140	0.8628	1.3707	1.3209	1.3249	0.8580
18	0.9018	0.9225	0.9140	0.8628	1.1473	1.4566	1.3535	0.8583
19	0.9018	0.9133	0.9232	0.8628	1.1239	1.2217	1.4786	0.8596
20	0.9018	0.9133	0.9140	0.8715	1.0196	1.0920	1.1416	0.8647
21	0.9018	0.9133	0.9049	0.8803	0.9523	1.0035	0.7640	0.8677
22	0.9018	0.9041	0.9140	0.8803	0.9218	0.6581	0.8306	0.8690
23	0.8928	0.9133	0.9140	0.8803	0.5884	0.7401	0.8682	0.8697
24	0.8928	0.9133	0.9049	0.8892	0.5521	0.6848	0.4665	0.8708
25	0.8928	0.9041	0.9140	0.8892	0.5449	0.2988	0.4749	0.8710

Optimal p^* allocation $= [0.8903, 0.9098, 0.9098, 0.8903]$, $\quad PR^* = 0.8712$

1.2.4 PSE Toolbox



1.2.5 Unimprovable allocation of WF

$$\widehat{PR}^* := \max_{p_1,\ldots,p_M;\prod_{i=1}^M p_i = p^*} \widehat{PR}(p_1,\ldots,p_M,N_1,\ldots,N_{M-1})$$

$$x(n+1) = (p^*)^{\frac{1}{M}} \prod_{i=1}^{M-1} \left(\frac{N_i + x(n)}{N_i + 1}\right)^{\frac{2}{M}}, \qquad x(0) \in (0,1)$$

Theorem: If $\sum_{i=1}^{M-1} N_i^{-1} \leq M/2$, then

$$x^* = \lim_{n\to\infty} x(n) = \widehat{PR}^* .$$

1.2.5 Unimprovable allocation of WF (cont)

Theorem: The unimprovable $p_1^*, \ldots, p_M^*, \prod_{i=1}^{M} p_i^* = p^*$, is given by

$$
\begin{aligned}
p_1^* &= \left(\frac{N_1+1}{N_1+\widehat{PR}^*}\right)\widehat{PR}^*, \\
p_i^* &= \left(\frac{N_{i-1}+1}{N_{i-1}+\widehat{PR}^*}\right)\left(\frac{N_i+1}{N_i+\widehat{PR}^*}\right)\widehat{PR}^*, i = 2, \ldots, M-1, \\
p_M^* &= \left(\frac{N_{M-1}+1}{N_{M-1}+\widehat{PR}^*}\right)\widehat{PR}^*.
\end{aligned}
$$

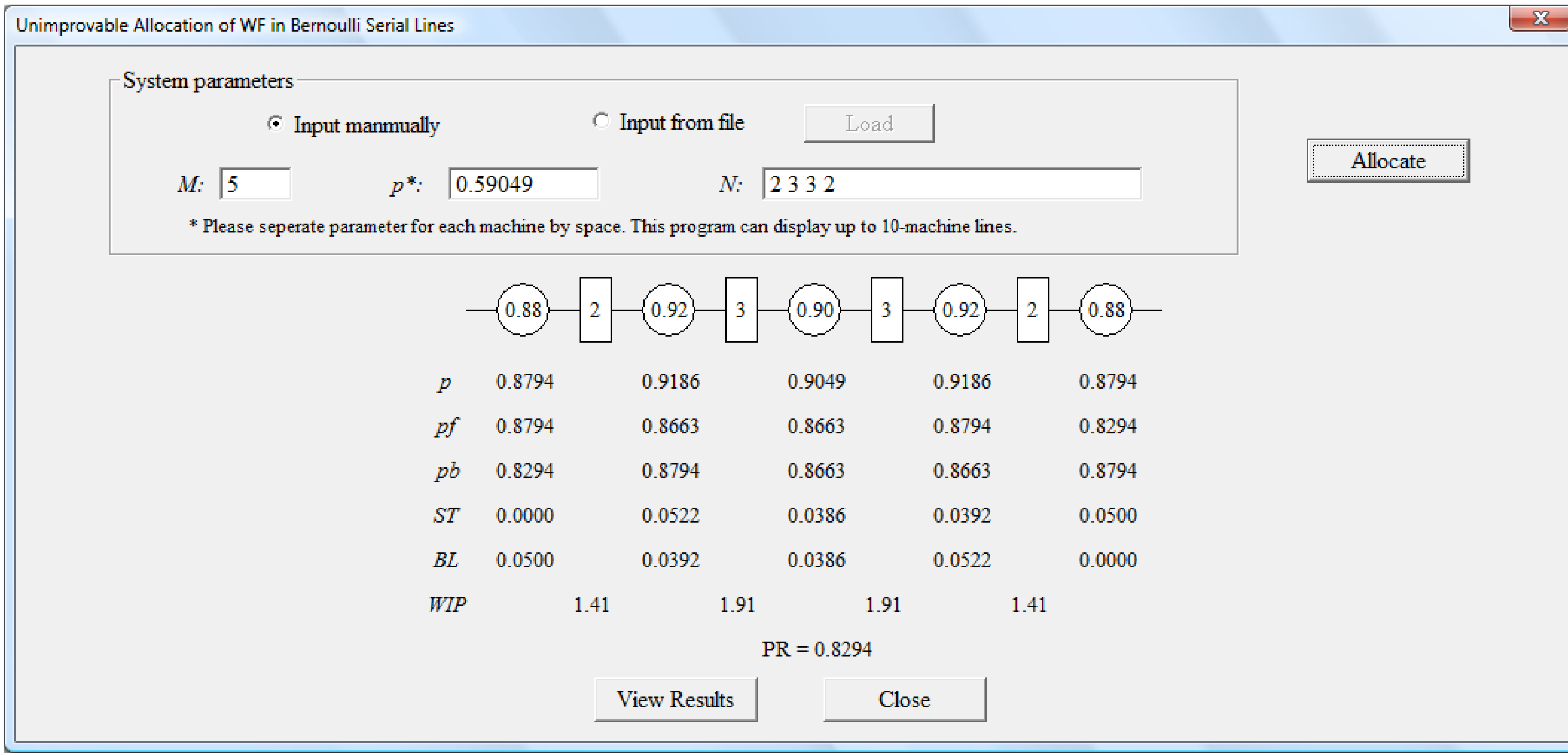
1.2.5 Unimprovable allocation of WF (cont)

Corollary: If $N_i := N$, $i = 1,\ldots, M$, unimprovable $p_1^*, \ldots, p_M^*, \prod_{i=1}^{M} p_i^* = p^*$ is a "flat" inverted bowl:

$$p_1^* = p_M^* < p_2^* = p_3^* = \ldots = p_{M-1}^*.$$

- In most cases, this allocation does not give a substantially larger $\widehat{PR}^*$ than the uniform allocation.

1.2.6 PSE Toolbox



1.3 Improvability w.r.t. WF and BC simultaneously

1.3.1 Necessary and sufficient conditions

Theorem: A Bernoulli line is unimprovable w.r.t. WF and BC simultaneously if and only if

$$p_1 = p_i^f = p_i^b = p_M, \qquad i = 2, \dots, M-1.$$

Corollary: Under this condition,

$$BL_i = ST_i,\ i = 2,\dots, M - 1,$$

$$BL_1 = ST_M.$$

In addition,

$$N_i = N \quad i = 1, \dots, M-1,$$

and

$$\widehat{WIP}_i = \frac{N(N+1)}{2(N+1-p_1)}$$

1.3.2 Unimprovable allocation of p_i and N_i

Denote

$$\widehat{PR}^{**} := \max_{\substack{N_1,\ldots,N_{M-1};\ \sum_{i=1}^{M-1} N_i = N^* \\ p_1,\ldots,p_M;\ \prod_{i=1}^{M} p_i = p^*}} \widehat{PR}(p_1,\ldots,p_M,N_1,\ldots,N_{M-1}).$$

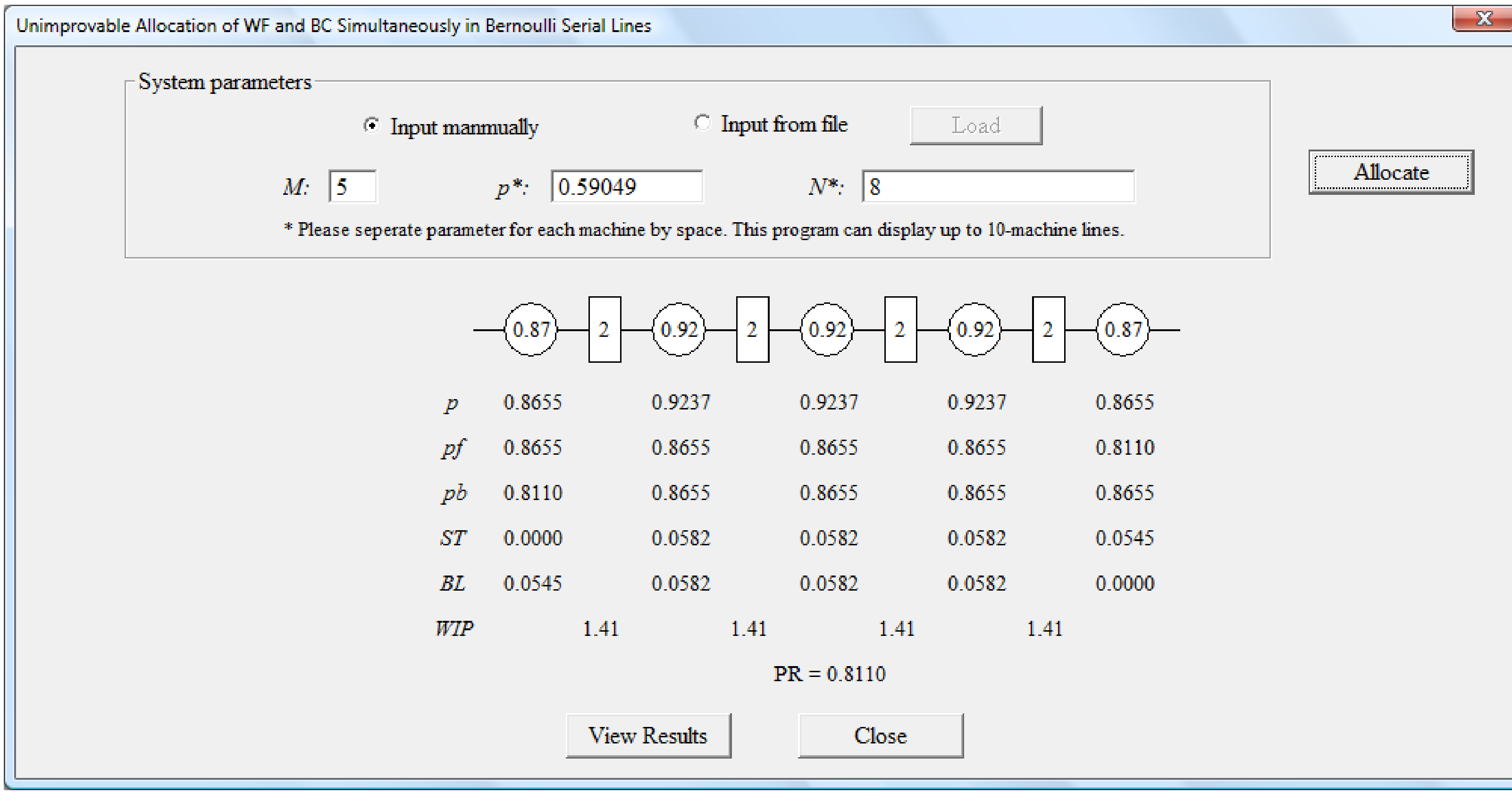
Theorem: The unimprovable sequence $p_1^*, \ldots, p_M^*$ and $N_1^*, \ldots, N_{M-1}^*$ are given by

$$N_i^* = \frac{N^*}{M-1} = N_{opt}^*, \qquad i = 1,\ldots,M-1,$$

$$p_1^* = p_M^* = \left(\frac{N_{opt}^*+1}{N_{opt}^*+\widehat{PR}^{**}}\right)\widehat{PR}^{**},$$

$$p_i^* = \left(\frac{N_{opt}^*+1}{N_{opt}^*+\widehat{PR}^{**}}\right)^2\widehat{PR}^{**}, \qquad i = 2,\ldots,M-1.$$

1.3.3 PSE Toolbox



1.4 Improvability w.r.t. BC

1.4.1 Necessary and sufficient conditions

Theorem: A Bernoulli line is unimprovable w.r.t. BC if and only if

$$\min_{i=1,\ldots,M} p_i \left(\min \left\{ \frac{p_i^f}{p_i^b}, \frac{p_i^b}{p_i^f} \right\} \right)$$

is maximized over all sequences $N_1', \ldots, N_{M-1}'$ such that

$$\sum_{i=1}^{M-1} N_i' = N^*.$$

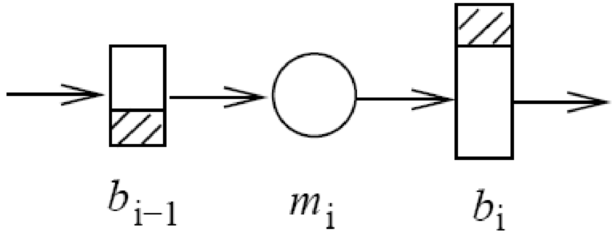
1.4.2 Practical equivalence statement

- **Numerical Fact:** The production rate under the above allocation is practically always the same as that for allocation that minimize

$$\max_{i=2,\ldots,M-1} |WIP_{i-1} - (N_i - WIP_i)|$$

over all sequences $N'_1, \ldots, N'_{M-1}$ such that $\sum_{i=1}^{M-1} N'_i = N^*$.

- "Common sense" justification of this fact:



1.4.3 BC improvability indicator and continuous improvement procedure

- **BC-Improvability Indicator:** A Bernoulli line is practically unimprovable w.r.t. BC if the average occupancy of each buffer is as close to the average availability of its downstream buffer as possible.

- **BC-Continuous Improvement Procedure:**

  (1) Evaluate WIP_i

  (2) Determine the buffer with the largest $|WIP_i - (N_{i+1} - WIP_{i+1})|$

  (3) Re-allocate N_i and N_{i+1}

  (4) Return to (1)

  (5) Continuous until arriving at a limit cycle and select the best allocation from the limit cycle

1.4.4 Example

- $M = 11$; $p_i = 0.8$, $i \neq 6$; $p_6 = 0.6$. Total buffer capacity 24.
- BC-Continuous Improvement Procedure results in

$$N_1 = 1, N_2 = N_3 = N_4 = 2, N_5 = 5,$$
$$N_6 = 4, N_7 = N_8 = N_9 = N_{10} = 2.$$

Resulting throughput $\widehat{PR} = 0.5843$.

- Goldratt's allocation

$$N_i = 1, i \neq 5,$$
$$N_5 = 17.$$

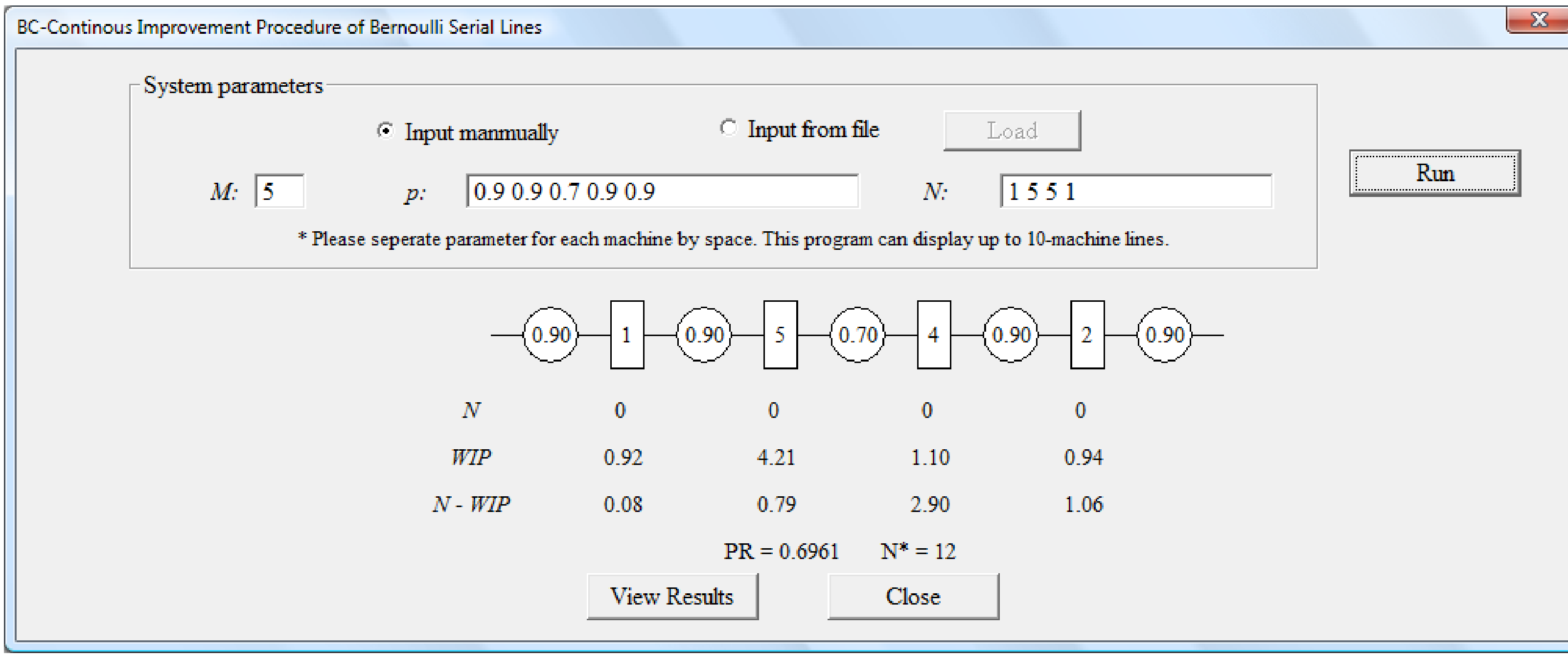
Resulting throughput $\widehat{PR} = 0.426$ (27% less).

- Improved Goldratt's allocation

$$N_i = 1, i \neq 5,6$$
$$N_5 = N_6 = 8.$$

Resulting throughput $\widehat{PR} = 0.491$ (16% less).

1.4.5 PSE Toolbox



2. Unconstrained Improvability

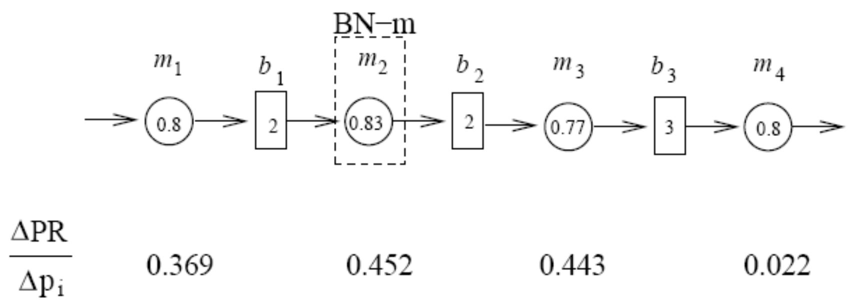
2.1 Bottleneck machine

Definition: m_i, $i \in \{1,\ldots, M\}$, is the *bottleneck machine* (BN-m) if

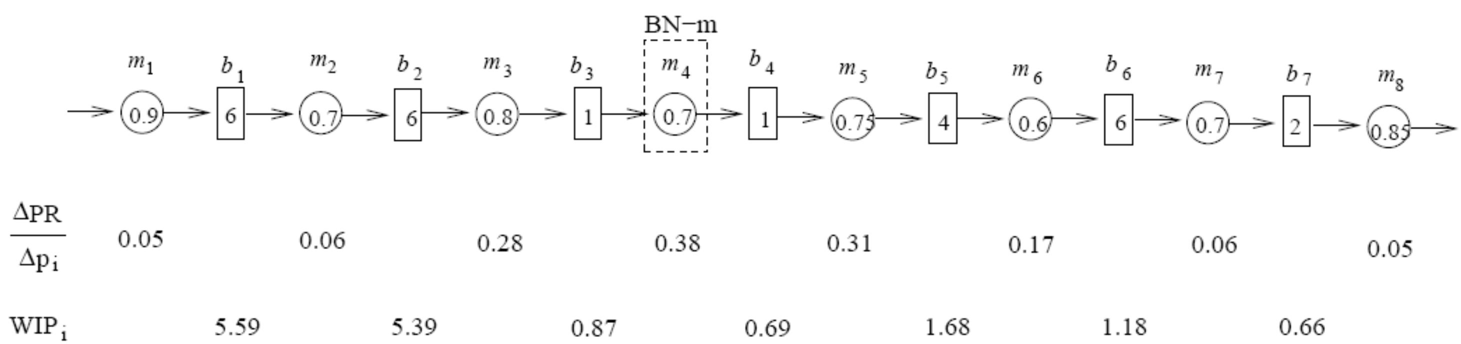
$$\frac{\partial PR}{\partial p_i} > \frac{\partial PR}{\partial p_j}, \qquad \forall j \neq i.$$

2.1 Bottleneck machine (cont)

- The worst machine is not necessarily the BN-m.
- The machine with the largest WIP is not necessarily the BN-m.



(a). The best machine is the bottleneck.



(b). The worst machine is not the bottleneck.

2.1 Bottleneck machine (cont)

Theorem: In WF-unimprovable Bernoulli lines,

$$\frac{\partial PR}{\partial p_i} p_i = const, \qquad i = 1, \ldots, M.$$

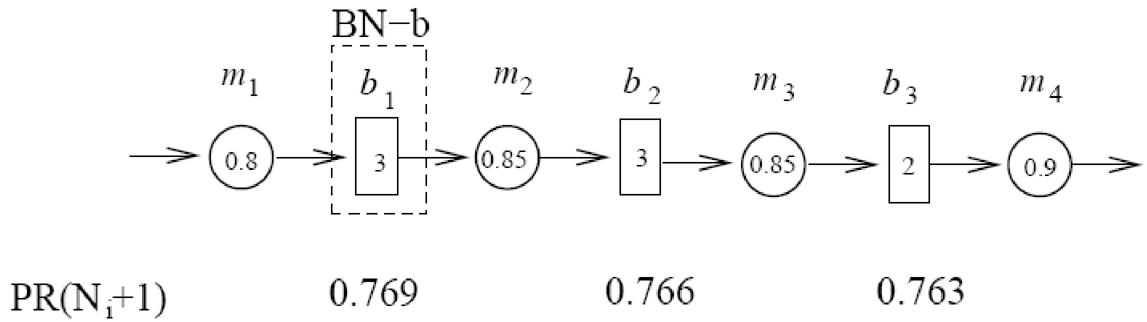
Thus, in a WF-unimprovable system, BN-m is necessarily the worst machine.

2.2 Bottleneck buffer

Definition: b_i, i = 1,…, M - 1, is the *bottleneck buffer* (BN-b) if

$$PR(p_1,\ldots,p_M,N_1,\ldots,N_i+1,\ldots,N_{M-1}) \\ > PR(p_1,\ldots,p_M,N_1,\ldots,N_j+1,\ldots,N_{M-1}), \quad \forall j \neq i.$$

- The smallest capacity buffer is not necessarily the BN-b.



2.3 Bottlenecks in two-machine lines

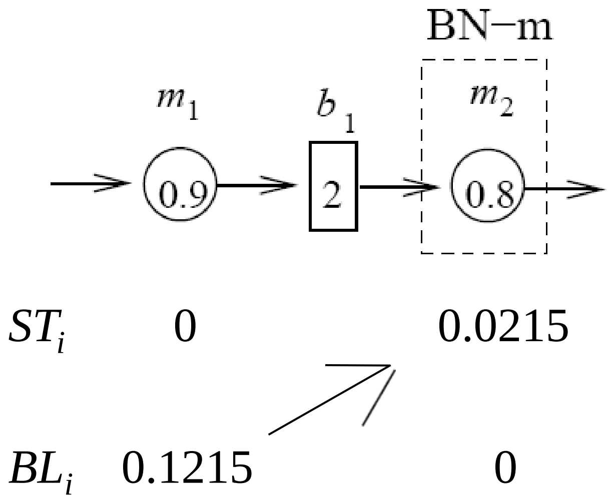
Theorem:

$$\frac{\partial PR}{\partial p_1} > \frac{\partial PR}{\partial p_2} \quad \text{if and only if} \quad BL_1 < ST_2 \,;$$

$$\frac{\partial PR}{\partial p_2} > \frac{\partial PR}{\partial p_1} \quad \text{if and only if} \quad BL_1 > ST_2 \,;$$

2.3 Bottlenecks in two-machine lines (cont.)

- **Illustration**

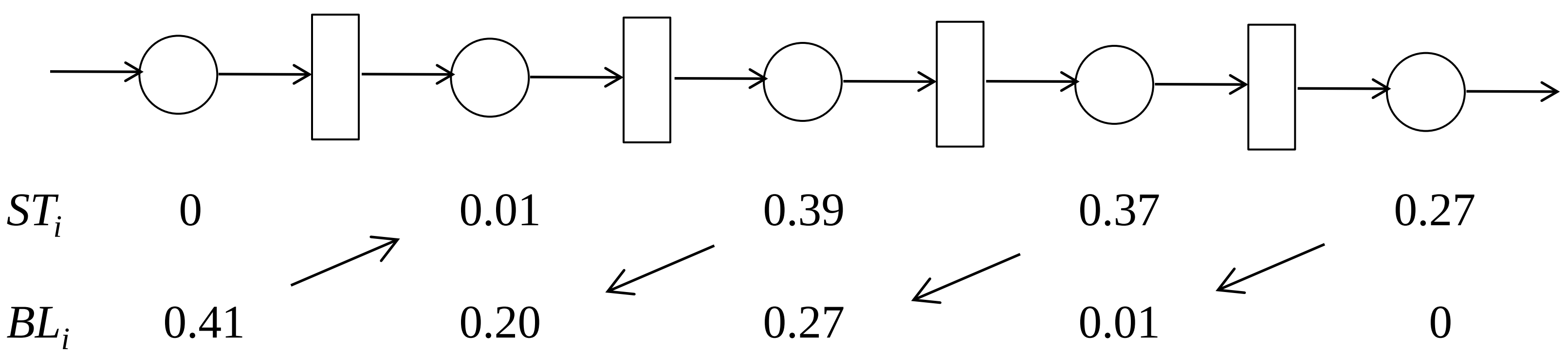


2.4 Bottlenecks in M > 2-machine lines

2.4.1 Arrow assignment rule:

$$BL_i > ST_{i+1} \rightarrow \text{arrow from } m_i \text{ to } m_{i+1}$$

$$BL_i < ST_{i+1} \rightarrow \text{arrow from } m_{i+1} \text{ to } m_i$$



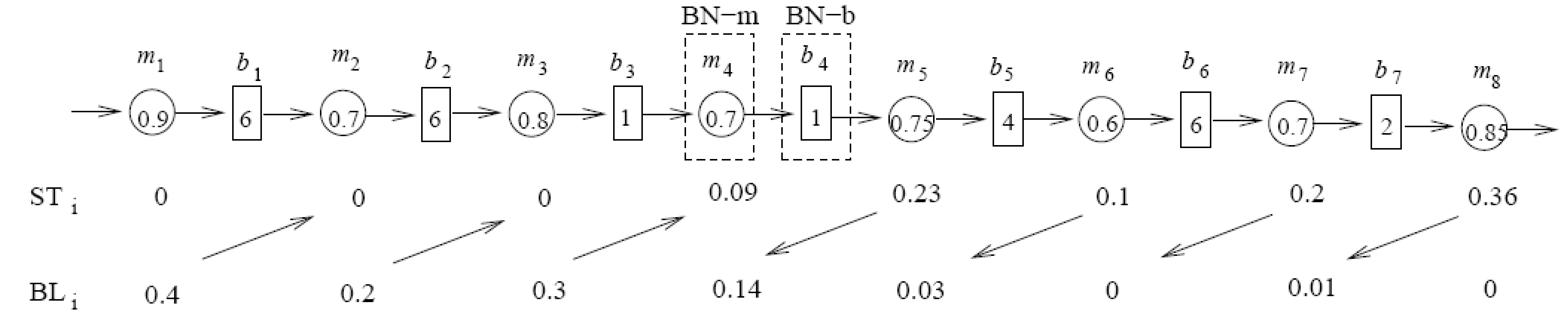
2.4.2 Bottleneck Indicator

- A single machine with no emanating arrows is the BN-m.
- Among multiple machines with no emanating arrows, the one with the largest severity is the Primary BN (PBN-m):

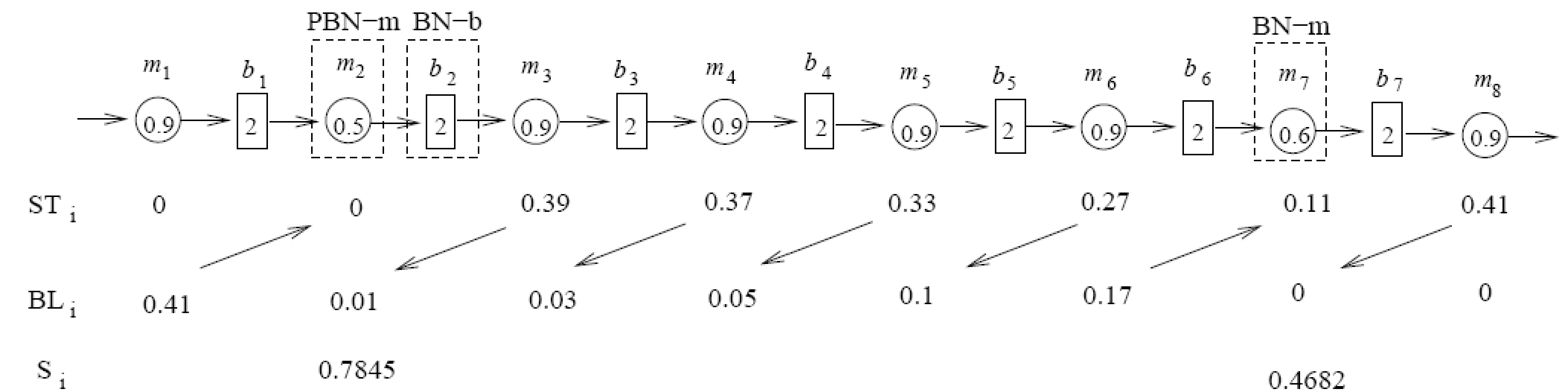
$$
\begin{aligned}
S_i &= |ST_{i+1} - BL_i| + |BL_{i-1} - ST_i|, \;\; i = 2, \ldots, M-1, \\
S_1 &= |ST_2 - BL_1|, \\
S_M &= |BL_{M-1} - ST_M|.
\end{aligned}
$$

- BN-b is the buffer in front of the BN-m if it is more often starved than blocked or right after the BN-m if it is more often blocked than starved

2.4.3 Illustrations



(a) Single bottleneck case

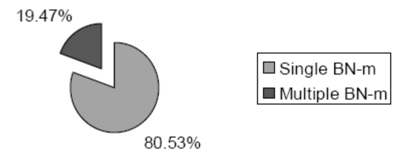


(b) Multiple bottleneck case

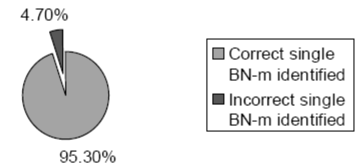
2.4.4 Numerical justification

- **Statistical experiment**
  - 5000 five-machine lines
  - $p_i \quad \in \quad \{0.75, 0.80, 0.85, 0.90.0.95\},$
  - $N_i \quad \in \quad \{1, 2, 3\}$

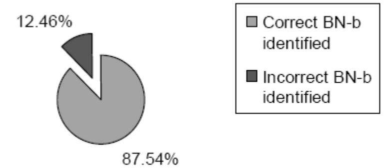
Results (using calculation data)



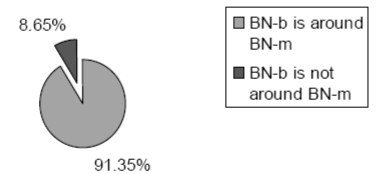
(a) Single vs. multiple BN-m



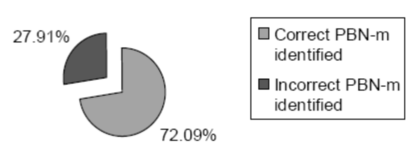
(b) Accuracy of BN-m identification in single BN-m case



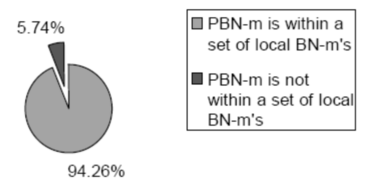
(c) Accuracy of BN-b identification in single BN-m case



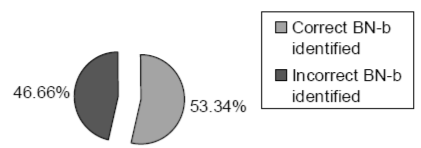
(d) BN-b is one of the buffers surrounding the BN-m in single BN-m case



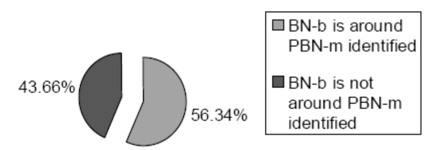
(e) Accuracy of PBN-m identification



(f) PBN-m is within the set of local BN-m's

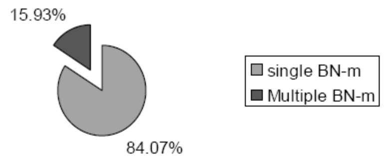


(g) Accuracy of BN-b identification in multiple BN-m case

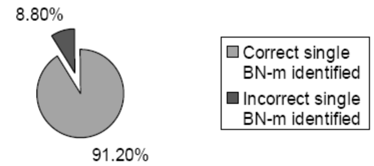


(h) BN-b is one of the buffers surrounding PBN-m

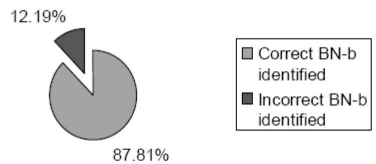
■ Results (using simulation data)



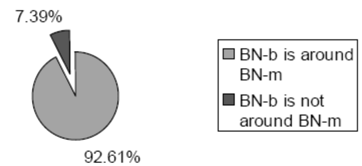
(a) Single vs. multiple BN-m



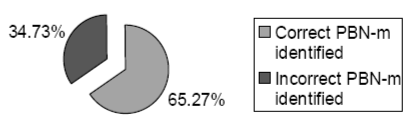
(b) Accuracy of BN-m identification in single BN-m case



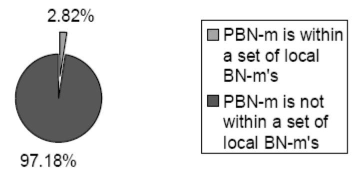
(c) Accuracy of BN-b identification in single BN-m case



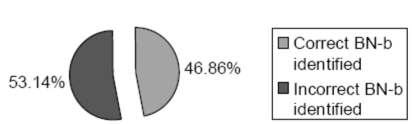
(d) BN-b is one of the buffers surrounding the BN-m in single BN-m case



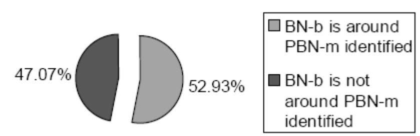
(e) Accuracy of PBN-m identification



(f) PBN-m is within the set of local BN-m's



(g) Accuracy of BN-b identification in multiple BN-m case



(h) BN-b is one of the buffers surrounding PBN-m

2.4.5 Analytical justification

- **Hypothesis:**

  $BL_{j-1} > ST_j$ implies $\epsilon_{j1} := P_{j-1}(0) \approx Q(p^f_{j-1}, p^b_j, N_{j-1}) \ll 1,$

  $BL_j < ST_{j+1}$ implies $\epsilon_{j2} := (1 - p^b_{j+1})P_j(N_j) \approx Q(p^b_{j+1}, p^f_j, N_j) \ll 1.$

- **Lemma:** For any $0 < \epsilon_0 \ll 1$, there exists N^*, such that if $N_j > N^*$, then

$$\epsilon = \max(\epsilon_{j1}, \epsilon_{j2}) < \epsilon_0.$$

- **Theorem:** Under the Hypothesis, BN-m is downstream of m_j if $\widehat{BL}_j > \widehat{ST}_{j+1}$ and upstream of m_j if $\widehat{BL}_{j-1} > \widehat{ST}_j$.

2.4.6 BN-Continuous Improvement Procedure

(1) Determine BL_i and ST_i.

(2) Identify BN-m and BN-b.

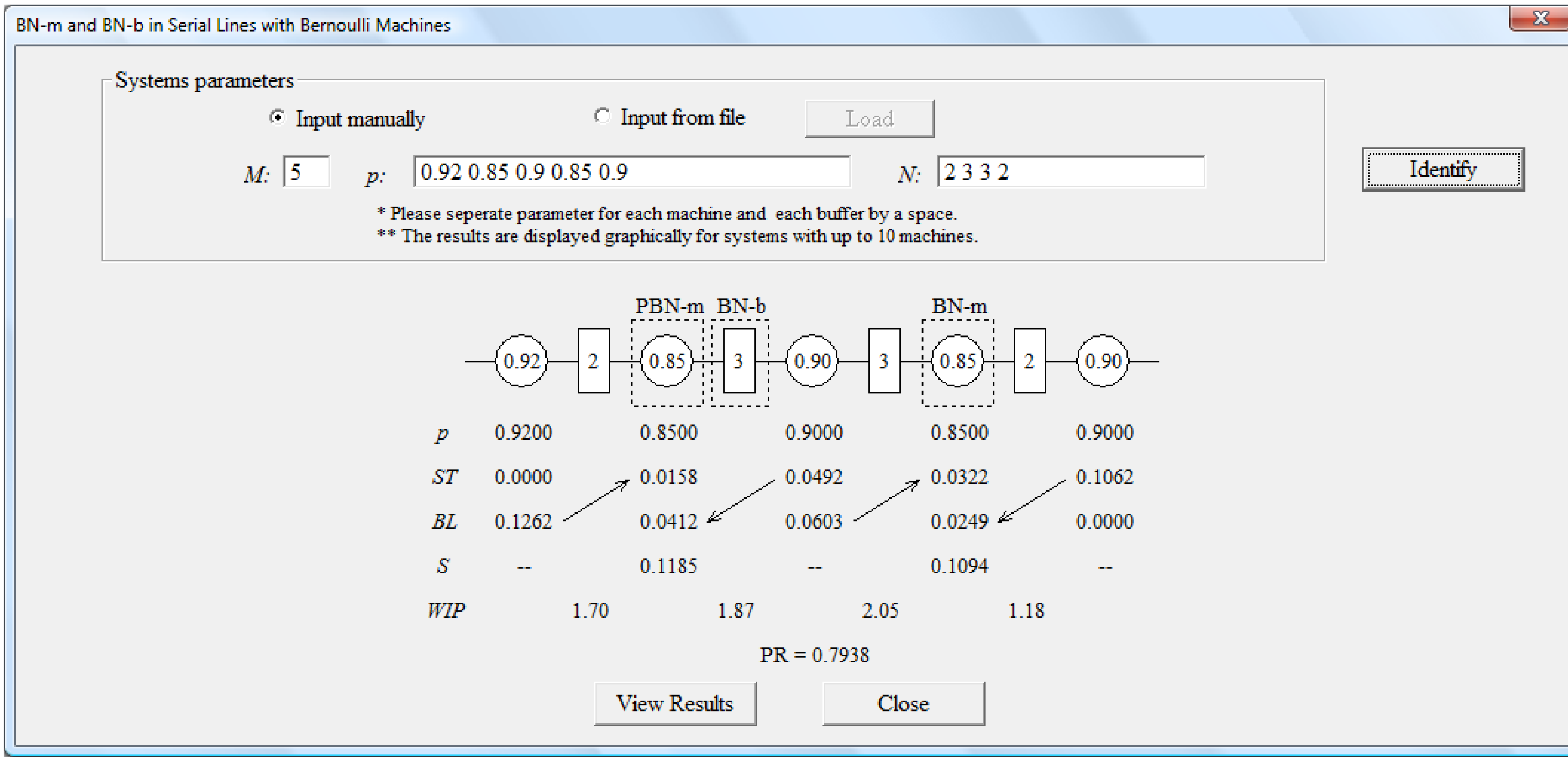
(3) Increase p_i of the BN-m.

(4) If impossible, increase N of the BN-b.

(5) Return to (1).

The most effective way for continuous improvement!

2.4.7 PSE Toolbox



2.5 Buffer potency

2.5.1 Definition

The buffering is

- *weakly potent* if BN-m is the worst one in the system;
- *potent* if it is weakly potent and $PR \approx 0.95p_i$(BN-m);
- *strongly potent* if it is potent and $N^* = \sum_{i=1}^{M-1} N_i$ is the smallest possible to ensure the desired production rate.

2.5.2 Illustration: Potency of automotive ignition module assembly system

- Analysis of production losses

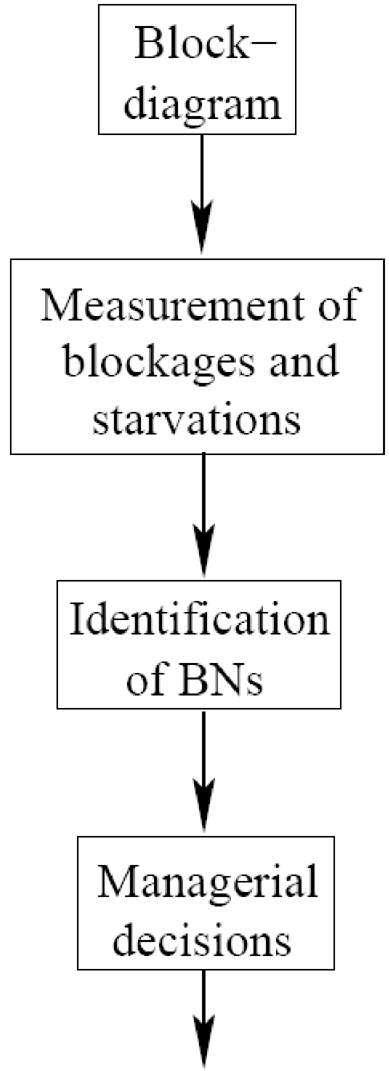
Month	May	June	July	Aug.	Sept.	Oct.
Isolation PR of the slowest machine (parts/h)	522	534	468	498	540	492
Losses due to machine (parts/h)	78	66	132	102	60	108
PR of the system (parts/h)	337	347	378	340	384	383
Losses due to MHS (parts/h)	185	187	90	158	156	109

- Observation: Out of roughly 240 parts/h lost, 80 parts/h are due to machines, while about 160 parts/h are due to MHS.
- Thus, ensuring MHS potency is an important resource of production systems improvements.

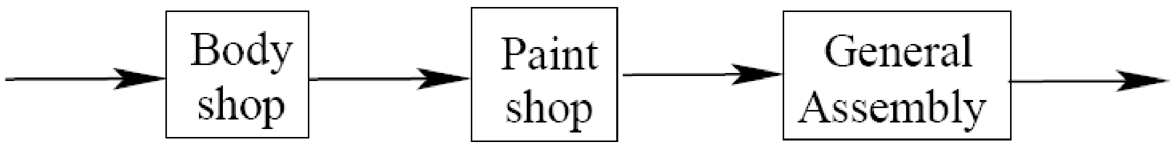
3. Measurement-Based Management

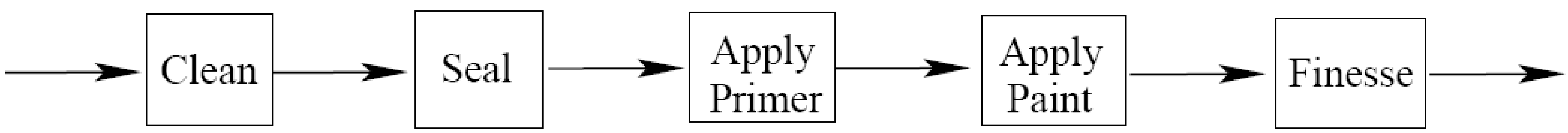
3.1 Purpose and structure

- Designing continuous improvement project requires relatively detailed information concerning the machines and buffers.
- A simpler method is needed for daily managerial duties.
- MBM is such a method. It consists of the following:


Block–diagram
Measurement of blockages and starvations
Identification of BNs
Managerial decisions

3.2 Simplified block diagrams



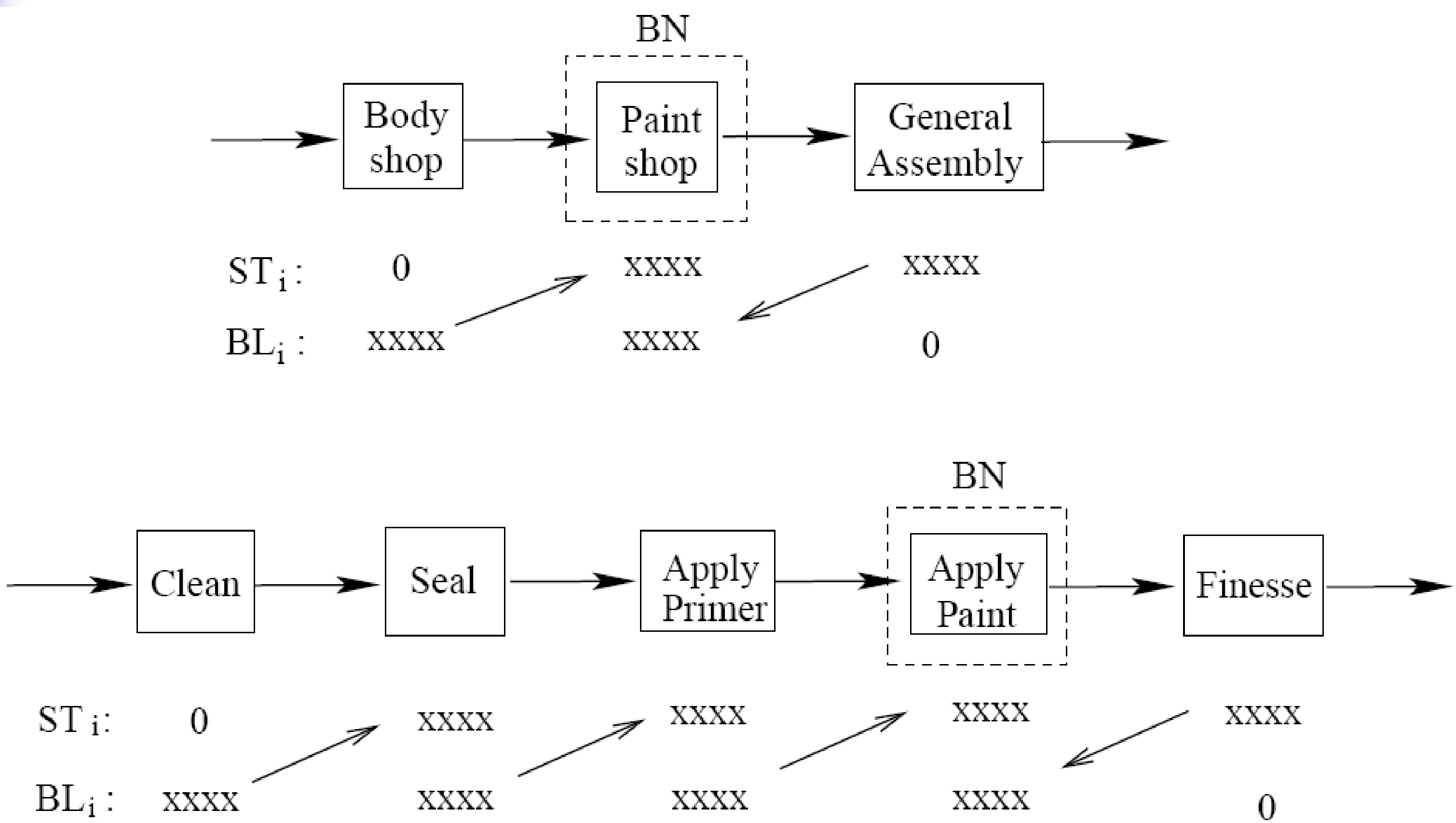


3.3 Measurement of blockages and starvations

$$BL_i = \frac{\text{Time of blockages of operation } i}{\text{Total time of observation}},$$

$$ST_i = \frac{\text{Time of starvations of operation } i}{\text{Total time of observation}}.$$

3.4 Identifying bottlenecks



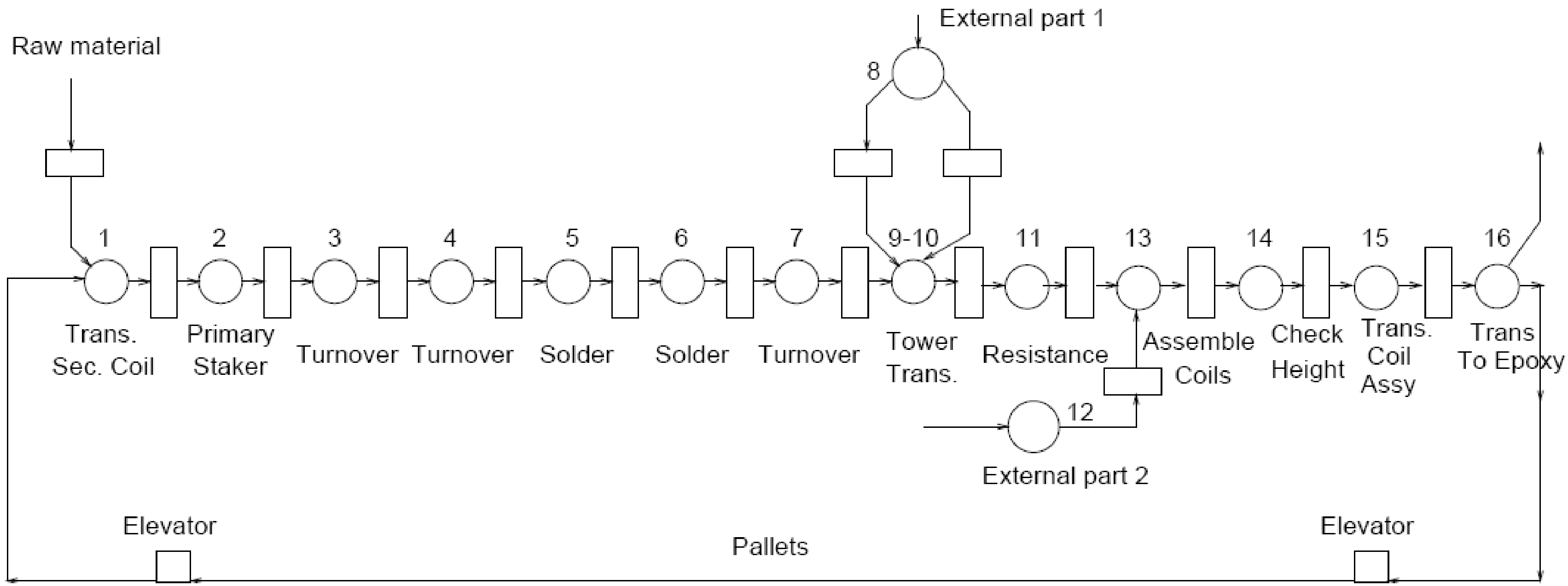
3.5 Managerial decisions

- Based on the previous steps, develop and plan activities to eliminate the bottlenecks identified.

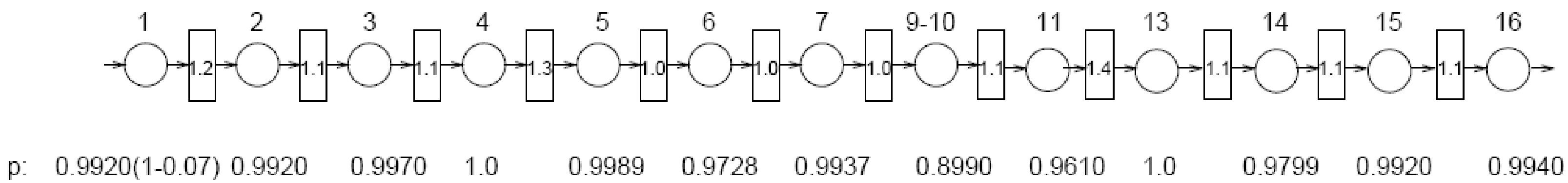
- Continue in the same manner in the never-ending process of continuous improvement.

4. Case Studies

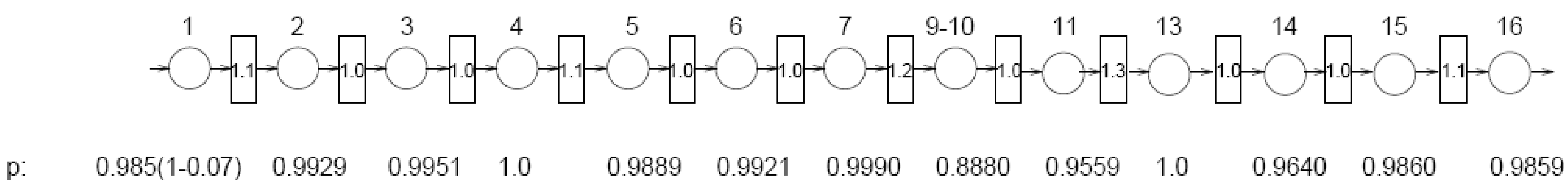
4.1 Automotive ignition coil processing system



4.1.1 Bernoulli model



(a). Period 1



(b). Period 2

4.1.2 Constrained improvability

- Constraints:

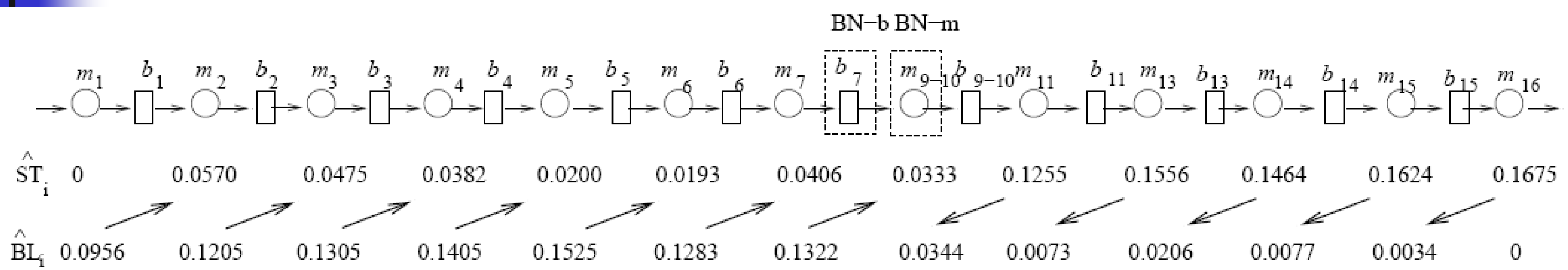
$$p^* = \prod_{i=1}^{M} p_i = 0.7312, \qquad N^* = \sum_{i=1}^{M-1} N_i = 13.2409.$$

- Unimprovable allocations

$$p_1^* = p_{16}^* = 0.9221, \qquad p_i^* = 0.9864, \qquad i = 2, \ldots, 15,$$
$$N_i^* = 1.1034, \qquad i = 1, \ldots, 15.$$
$$\widehat{PR} = 0.8580$$

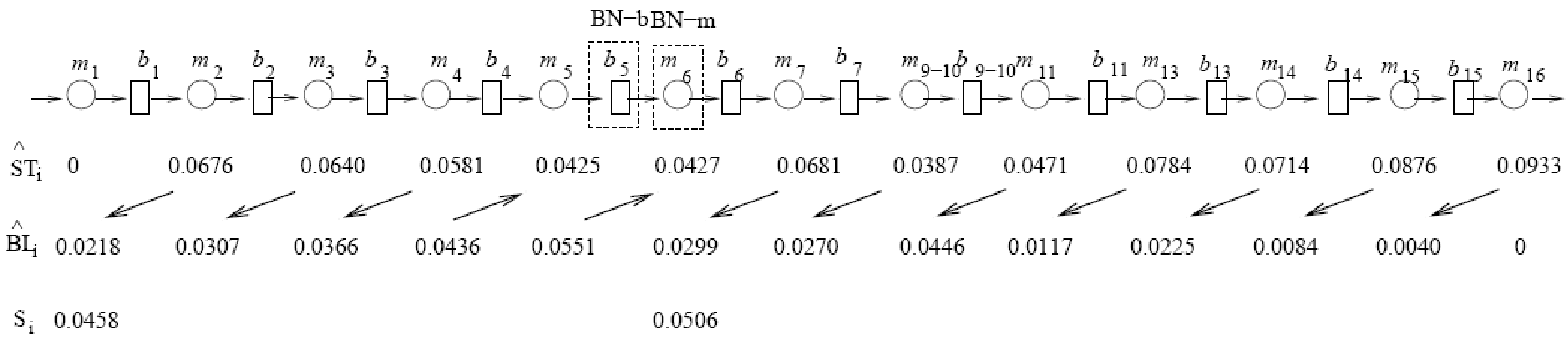
- Observation: *PR* is still low, i.e., constrained improvability does not solve the problem.

4.1.3 Unconstrained improvability



- Increasing BN-b by 1 and $p_{9\text{-}10}$ by 10% lead to $\widehat{PR} = 0.8976$ ($\widehat{TP} = 505$ parts/hour).
- Observation: *PR* is still low.

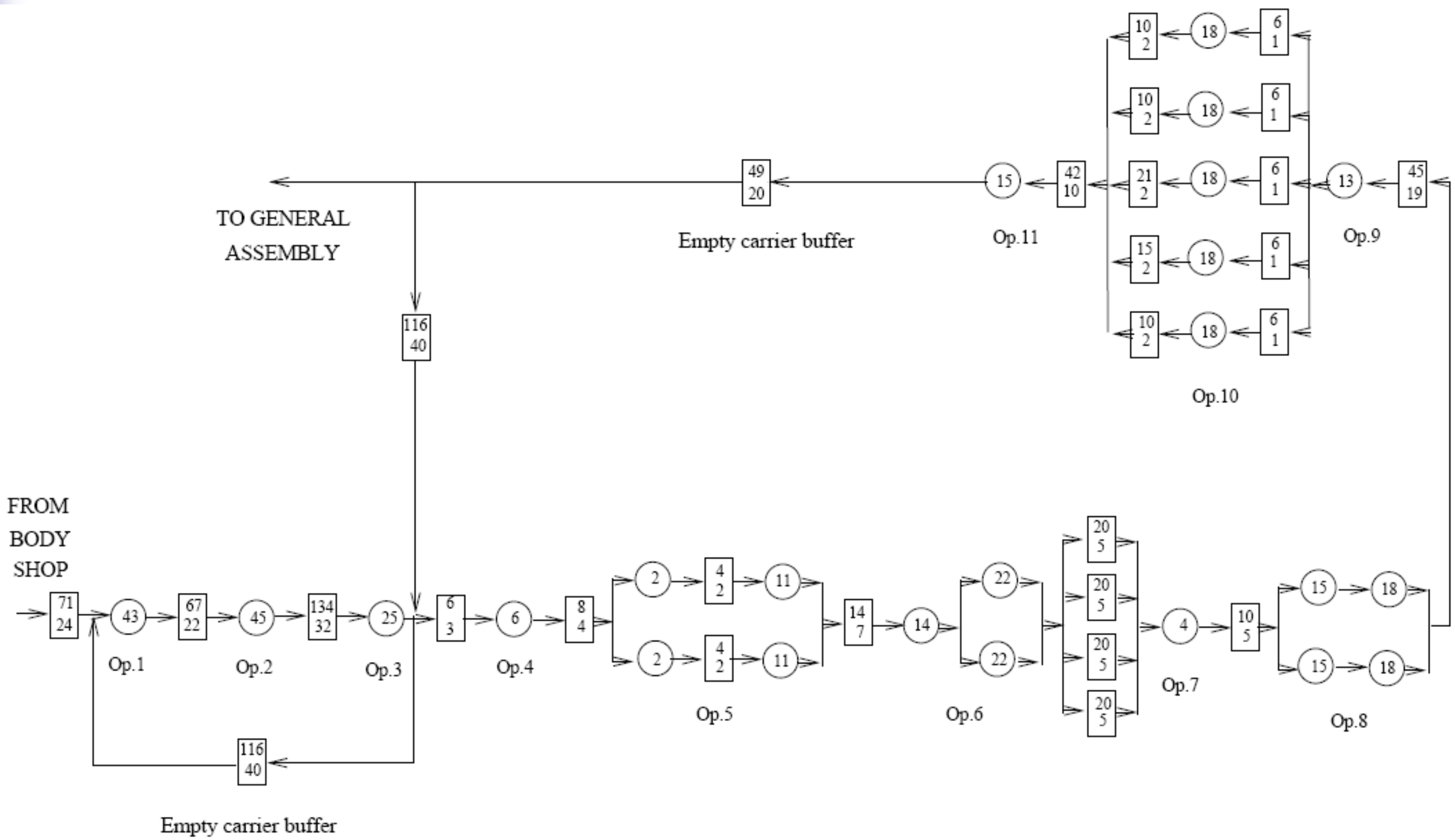
4.1.3 Unconstrained improvability (cont)



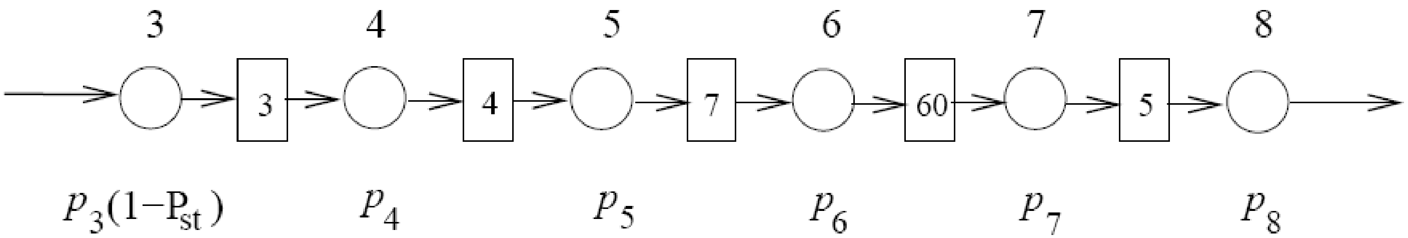
- Increasing b_5 by 1 leads to $\widehat{PR} = 0.9092$ ($\widehat{TP} = 511.4$ parts/hour), which has been viewed as acceptable performance
- Returning to exponential model (using B-exp transformation):

$$T^{tr}_{up,9-10} = 123.11 \text{ min}, \qquad N^{tr}_5 = 17, \qquad N^{tr}_7 = 10.$$

4.2 Automotive paint shop production system



4.2.1 Bernoulli model



4.2.2 Constrained improvability

- Constraints:

$$p^* = \prod_{i=1}^{M} p_i = 0.7877, \qquad N^* = \sum_{i=1}^{M-1} N_i = 79.$$

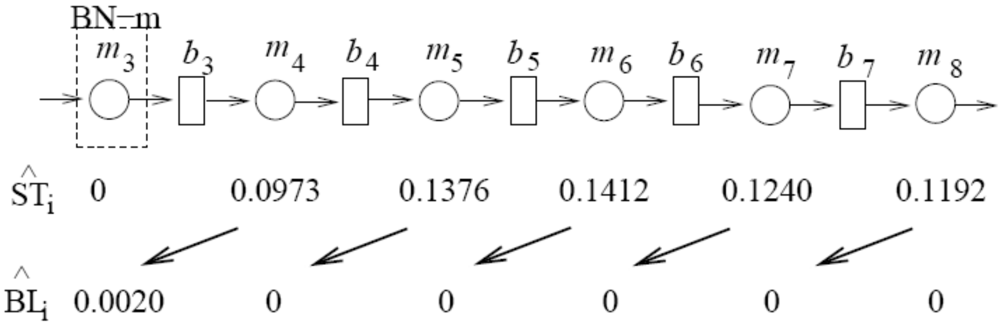
- Unimprovable allocation:

$$p_3^* = p_8^* = 0.9594, \quad p_i^* = 0.9618, \quad i = 4, \ldots, 7,$$

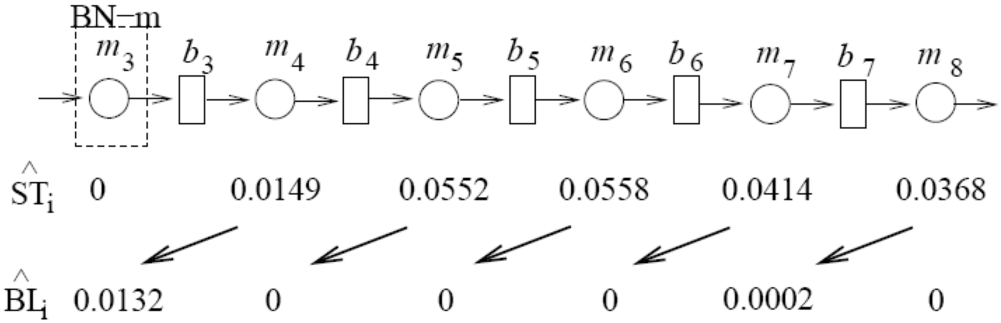
$$N_i^* = 15.8, \qquad i = 3, \ldots, 7.$$

- After some adjustment, $\widehat{PR} = 0.9119$ ($\widehat{TP}$ = 57.45 parts/hour).
- Observation: *PR* is relatively low.

4.2.3 Unconstrained improvability

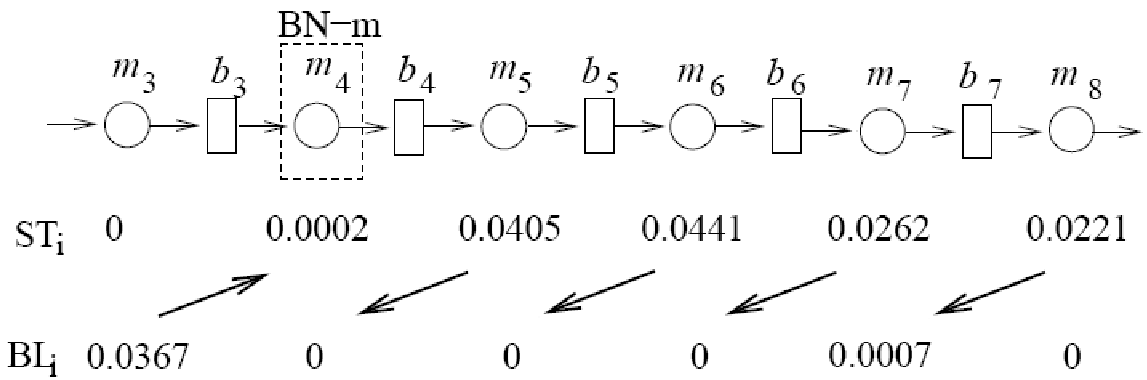


- Assuming that there are no starvations for skids, $\widehat{PR} = 0.9411$ ($\widehat{TP} = 59.29$ jobs/hour).



4.2.3 Unconstrained improvability (cont)

- Increasing efficiency of m_3 by 4% results in



$$\widehat{PR} = 0.9558 \ (\widehat{TP} = 60.22 \text{ jobs/hour})$$

5. Summary

- Improvements can be accomplished in either constrained or unconstrained scenarios.
- WF-unimprovable system: buffers close to being half full.
- WF- and BC-unimprovable system: blockages and starvations of all “internal” machines are the same.
- BC-unimprovable system: each machine is blocked and starved with almost equal frequencies.
- BN-m and BN-b can be identified using the arrow-based rule.
- Buffer is potent if the worst machine is indeed the BN.